Zeeman effect implementation in ARTS

Nikolay Koulev

Institute for Environmental Physics
nkoulev@uni-bremen.de
http://www.sat.uni-bremen.de

Bredbeck
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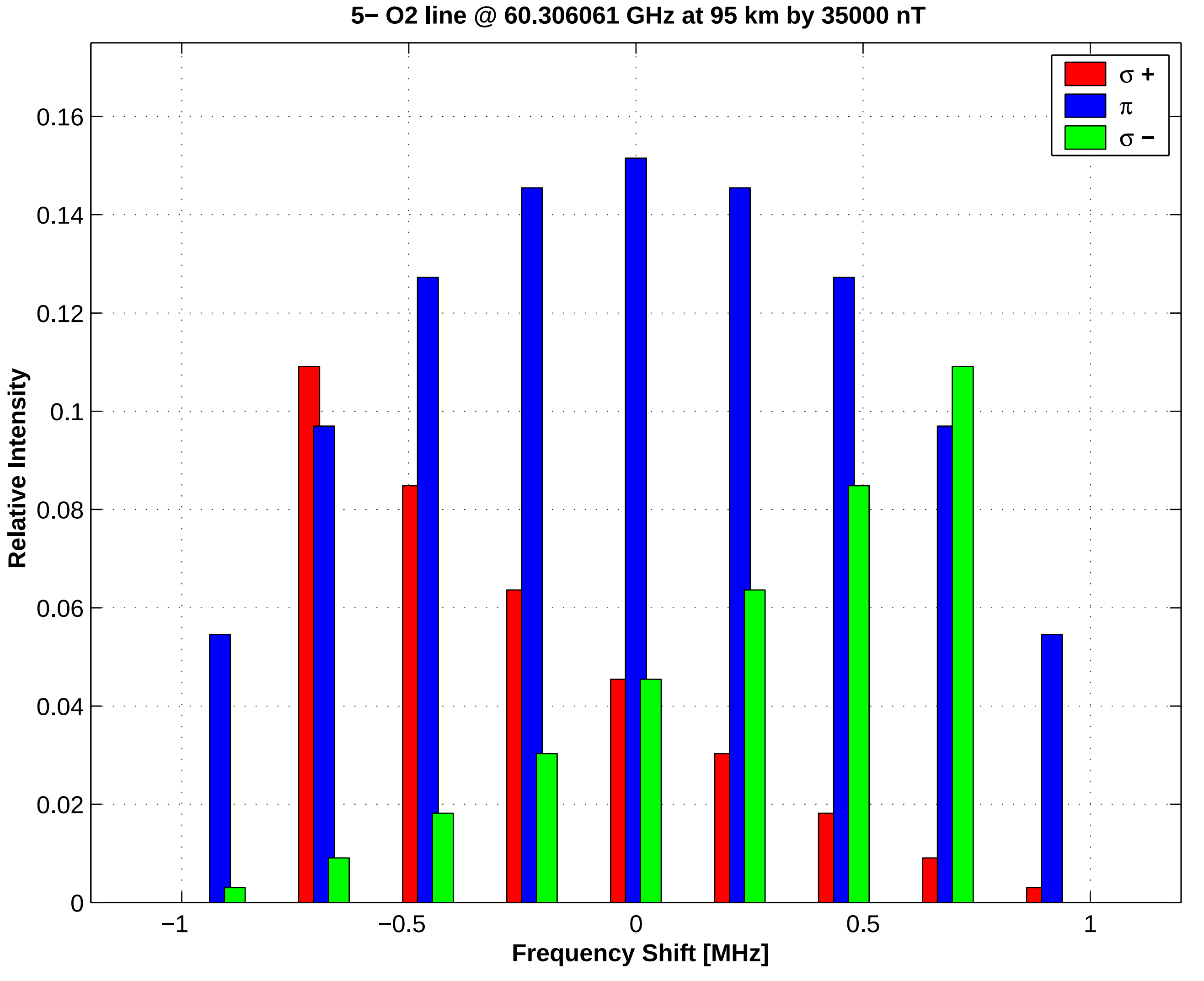
Zeeman effect



Non-scattering Vector RTE in ARTS

$$\frac{d}{ds}\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu) = -\hat{\mathbf{K}}(\vec{\mathbf{n}},\nu)\cdot\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu) + \vec{\mathbf{a}}(\vec{\mathbf{n}},\nu)B(\nu)$$

- $\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu)$ - 4D Stokes vector
- $\hat{\mathbf{K}}(\vec{\mathbf{n}},\nu)$ - 4x4 Extinction coefficient matrix
- $\vec{\mathbf{a}}(\vec{\mathbf{n}},\nu)$ - 4D Absorption coefficient vector
- $\vec{\mathbf{n}}$ - propagation vector
- $B(\nu)$ - Planck function

Zeeman effect RTE

$$\frac{d}{ds}\hat{\mathbf{R}}(\vec{\mathbf{n}},\nu) = -[\hat{\mathbf{G}}(\nu)\cdot\hat{\mathbf{R}}(\vec{\mathbf{n}},\nu)+\hat{\mathbf{R}}(\vec{\mathbf{n}},\nu)\cdot\hat{\mathbf{G}}^*(\nu)] + B(\nu)[\hat{\mathbf{G}}(\nu)+\hat{\mathbf{G}}^*(\nu)]$$

- $\hat{\mathbf{R}}(\vec{\mathbf{n}},\nu)$ - 2x2 Radiation Intensity (Brightness temperature*) Matrix
- $\hat{\mathbf{G}}(\nu)$ - 2x2 complex Propagation tensor

$$\hat{\mathbf{G}}(\nu) = i\,\frac{2\,\pi\,\nu}{c}[\hat{\mathbf{I}}+\hat{\mathbf{M}}_{AN}(\nu)]$$

*then using the medium physical temperature T instead of $B(\nu)$

ARTS Zeeman effect approach

$$\frac{d}{ds}\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu) = -\hat{\mathbf{K}}_Z(\vec{\mathbf{n}},\nu) \cdot \vec{\mathbf{I}}(\vec{\mathbf{n}},\nu) + \vec{\mathbf{a}}_Z(\vec{\mathbf{n}},\nu)B(\nu)$$

where

$$\begin{aligned}\hat{\mathbf{K}}_Z(\vec{\mathbf{n}},\nu) &= \hat{f}(\hat{\mathbf{G}}(\nu)) \\ \vec{\mathbf{a}}_Z(\vec{\mathbf{n}},\nu) &= \vec{g}(\hat{\mathbf{G}}(\nu))\end{aligned}$$

Radiation matrices

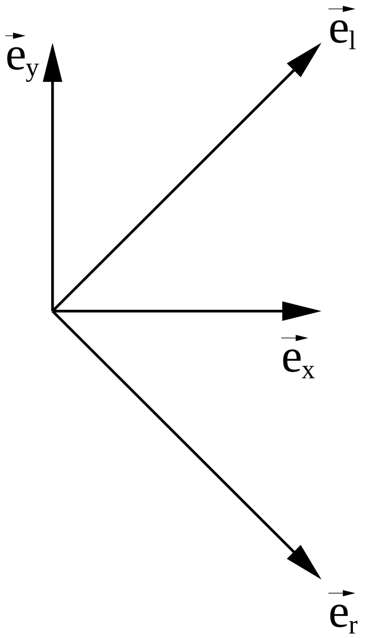
$$\hat{\mathbf{R}}(\vec{\mathbf{n}},\nu) \quad \sim \quad \hat{\mathbf{G}}(\nu)\cdot\left\langle \vec{\mathbf{H}}(\vec{\mathbf{n}},\nu)\otimes\vec{\mathbf{H}}^{*}(\vec{\mathbf{n}},\nu)\right\rangle$$

- $\vec{\mathbf{H}}(\vec{\mathbf{n}},\nu)$ - 2D complex field vector of the propagated wave.

$$\hat{\mathbf{C}}(\vec{\mathbf{n}},\nu) = \left\langle \vec{\mathbf{H}}(\vec{\mathbf{n}},\nu)\otimes\vec{\mathbf{H}}^{*}(\vec{\mathbf{n}},\nu)\right\rangle$$

- $\hat{\mathbf{C}}(\vec{\mathbf{n}},\nu)$ - complex Coherency matrix of the propagated wave.

Polarization bases



- $\vec{\mathbf{e}}_x$, $\vec{\mathbf{e}}_y$ - linear polarization basis.
- $\vec{\mathbf{e}}_l$, $\vec{\mathbf{e}}_r$ - circular polarization basis.

$$\begin{pmatrix} \vec{\mathbf{e}}_l \\ \vec{\mathbf{e}}_r \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \cdot \begin{pmatrix} \vec{\mathbf{e}}_x \\ \vec{\mathbf{e}}_y \end{pmatrix}$$

Coherency Matrix and Stokes Vector

In an arbitrary $(\vec{\mathbf{e}}_a, \vec{\mathbf{e}}_b)$ polarization basis:

$$\hat{\mathbf{C}}^{(ab)} = \frac{1}{2} \sum_{i=0}^{3} \mathbf{I}_i^{(ab)} \hat{\sigma}_i$$

where

$$\hat{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \hat{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \hat{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

and $\mathbf{I}_i^{(ab)}$ are the Stokes vector components.

Solution for $\hat{\mathbf{K}}_Z(\vec{\mathbf{n}},\nu)$ and $\vec{\mathbf{a}}_Z(\vec{\mathbf{n}},\nu)$

In a circular $(\vec{\mathbf{e}}_l, \vec{\mathbf{e}}_r)$ polarization basis:

$$\hat{\mathbf{K}}_Z^C = \frac{1}{2}\begin{pmatrix} A & B & C & 0 \\ B & A & 0 & -D \\ C & 0 & A & E \\ 0 & D & -E & A \end{pmatrix} \qquad \vec{\mathbf{a}}_Z^C = \begin{pmatrix} A \\ B \\ C \\ 0 \end{pmatrix}$$

where $A = 2Re[G_{11} + G_{22}]$, $B = -2Re[G_{22} - G_{11}]$, $C = 2Re[G_{12} + G_{21}]$, $D = 2Im[G_{12} + G_{21}]$ and $E = -2Im[G_{22} - G_{11}]$.

Stokes vector transformation

$$\vec{\mathbf{I}}_C(\vec{\mathbf{n}},\nu) = \hat{\mathbf{V}}_C \cdot \vec{\mathbf{I}}(\vec{\mathbf{n}},\nu)$$

where

$$\hat{\mathbf{V}}_C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

and $\vec{\mathbf{I}}_C(\vec{\mathbf{n}},\nu)$, $\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu)$ - Stokes vector in cirular and linear polarization basis.

ARTS final Zeeman effect RTE

In a linear $(\vec{\mathbf{e}}_x, \vec{\mathbf{e}}_y)$ polarization basis:

$$\frac{d}{ds}\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu) = -\overbrace{\hat{\mathbf{V}}_C^{-1}\cdot\hat{\mathbf{K}}_Z^C(\vec{\mathbf{n}},\nu)\cdot\hat{\mathbf{V}}_C}^{\hat{\mathbf{K}}_Z(\vec{\mathbf{n}},\nu)}\cdot\vec{\mathbf{I}}(\vec{\mathbf{n}},\nu) + \underbrace{\hat{\mathbf{V}}_C^{-1}\cdot\vec{\mathbf{a}}_Z^C(\vec{\mathbf{n}},\nu)}_{\vec{\mathbf{a}}_Z(\vec{\mathbf{n}},\nu)}B(\nu)$$

Additional implementation issues

The fully fledged Zeeman part in ARTS will utilize:

- all the pressure and temperature profiles provided by `arts-data` and `arts-xml-data` packages
- the IGRF2000 profiles of the global magnetic field for Epoch 2000-2005 with possibility of an update
- (possibly) the line-mixing mechanism between the split lines

References

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